Tilings: arts and math

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Math 108: Geometric Combinatorics

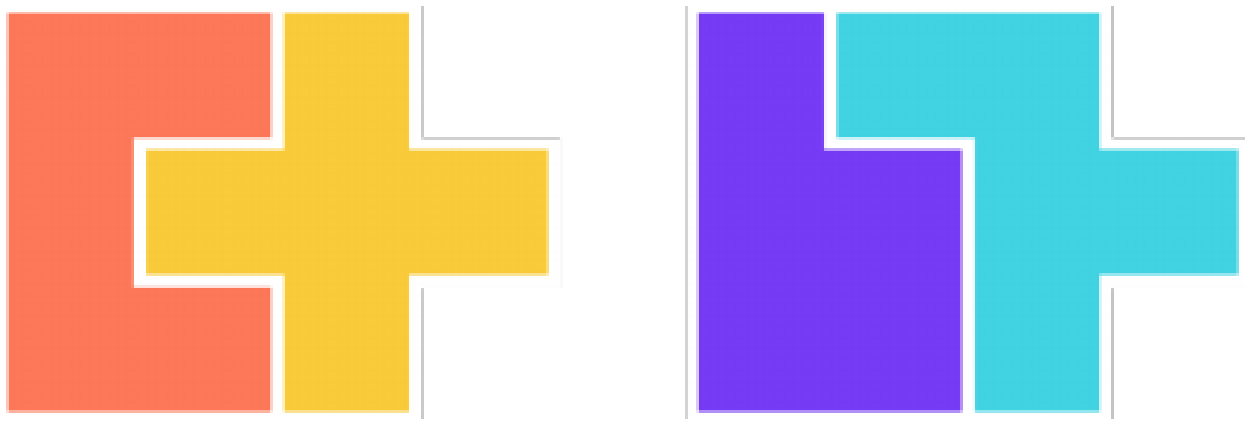
February 10, 2023

Solutions to the activity

Question 1: Many pairs work, including this one

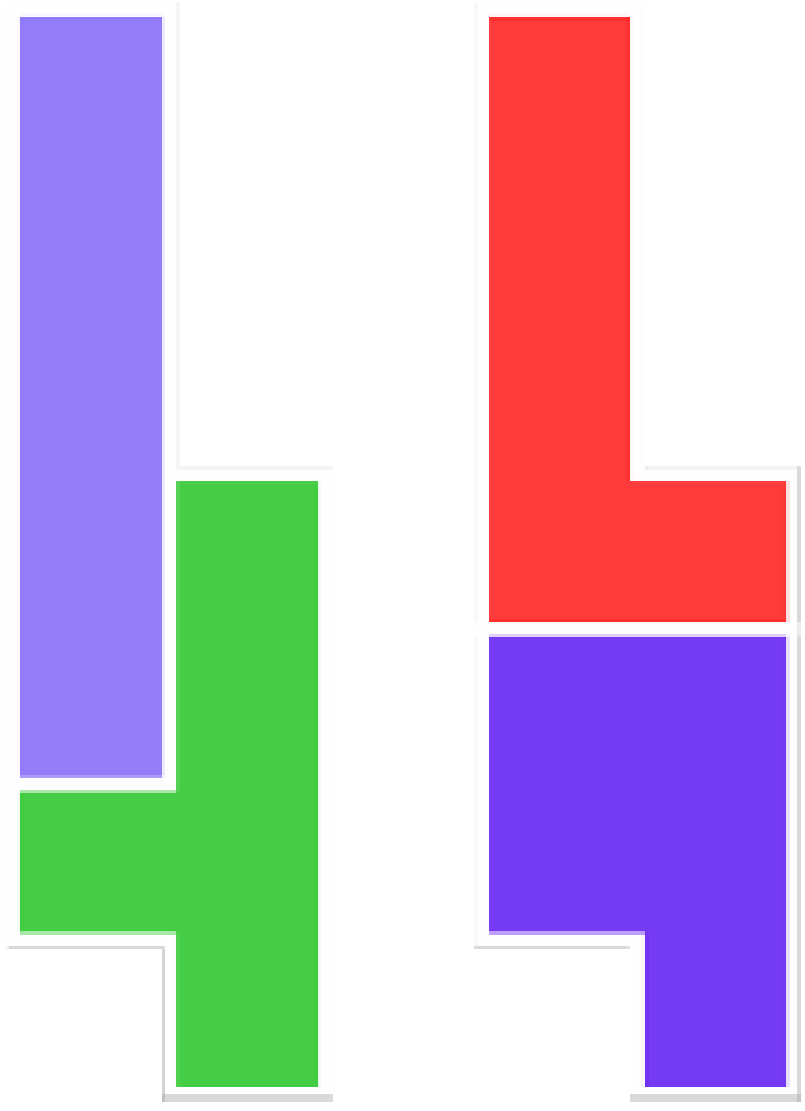
Solutions to the activity

Question 2: All solutions must involve the "P" (purple below), or the cross (yellow below)! Only four solutions.



Solutions to the activity

Question 3: There is only one solution!



Solutions to the activity

Question 4: There is only one solution!

Solutions to the activity

Question 5: There is only one solution (up to internal rearrangement within each tiling)

Solutions to the activity

Question 6: There are five solutions total to make 2 shapes (it is not possible to make a third). Here is a solution:

Solutions to the activity

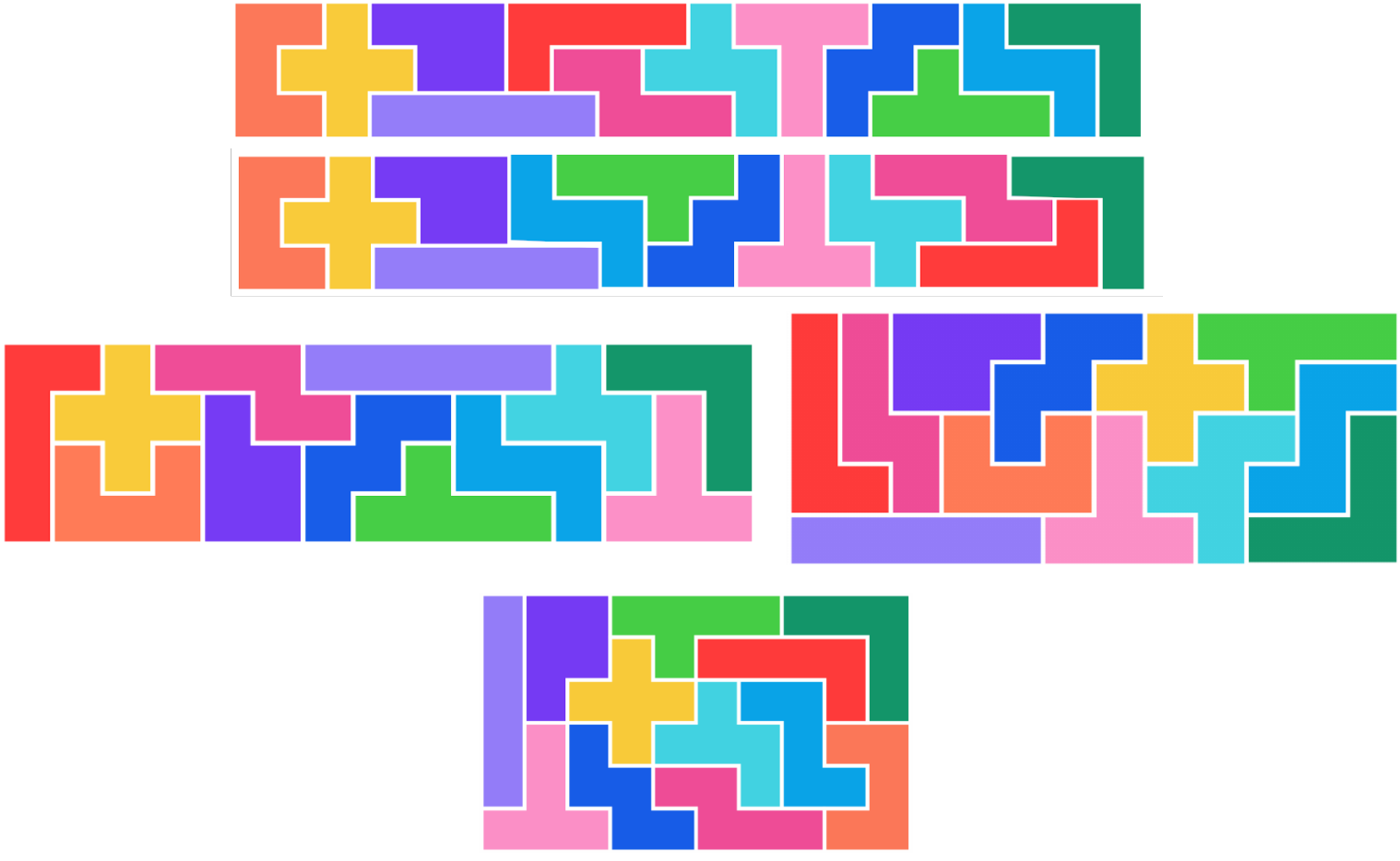
Question 7: There are 11 solutions total to make 2 shapes (it is not possible to make a third). Here is a solution:

Solutions to the activity

Question 8: Only one solution!

Solutions to the activity

Question 9: only 2 solutions for 3×20 (up to symmetry), but 2339 solutions for 6×10.



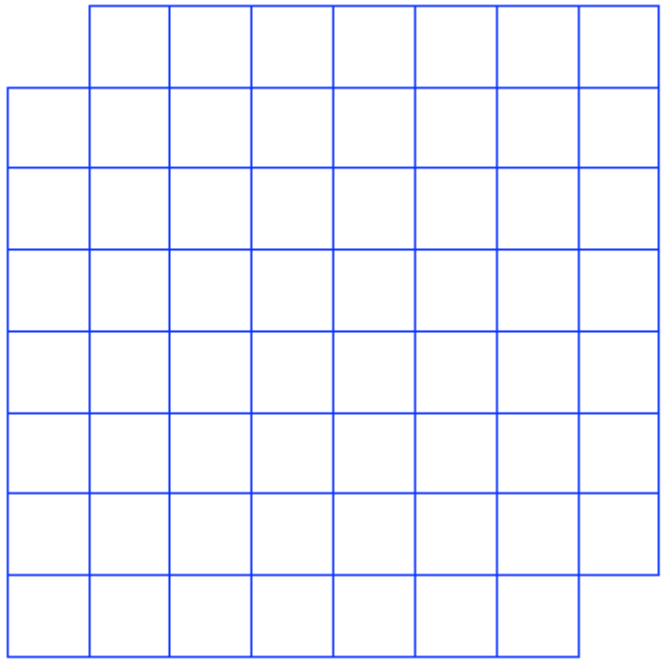
Questions about tilings: Given a fixed set of tiles, can we tile a fixed shape with it...

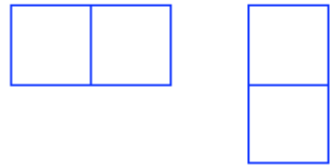
- ... allowing the tiles to be translated only?
- ... allowing the tiles to be translated or rotated?
- ... allowing the tiles to be translated, rotated or flipped?
- ... allowing multiple copies of the tiles?
- ... with a single tile, but an unlimited number of copies?
- ... tiling a bounded region?
- ... tiling the whole plane?

If there exists a tiling with these constraints, is the solution unique? How many solutions are there?

Tiling a fixed area

Can you tile an 8×8 chessboard with dominoes if corners are missing?





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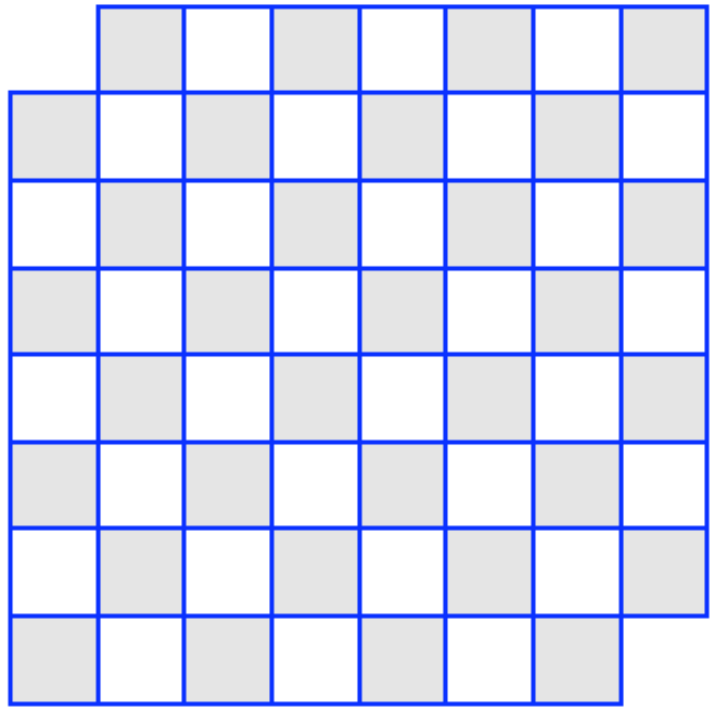
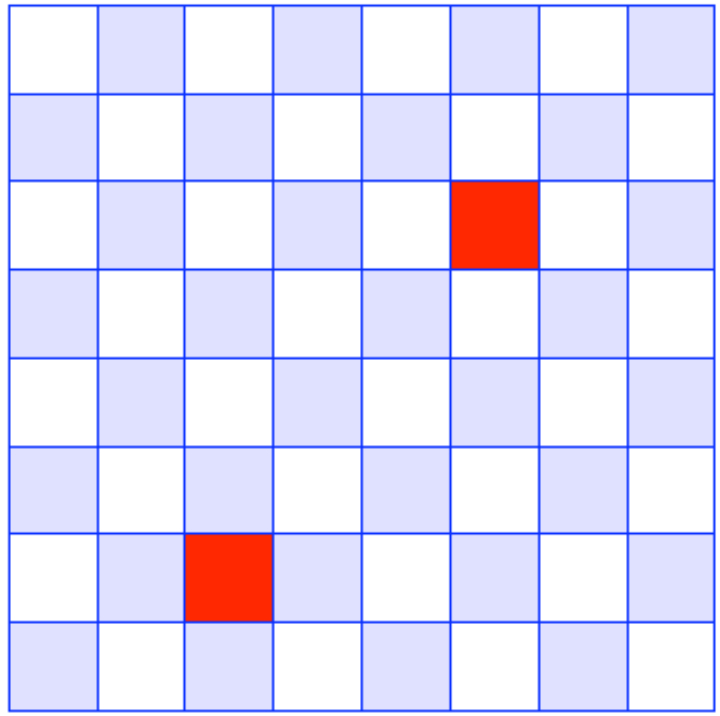


No, because a domino covers a black and a white cell, and the number of white cell is not the same as the number of black cells.

Tiling a fixed area

What if the two tiles have the same color?



Tiling a fixed area

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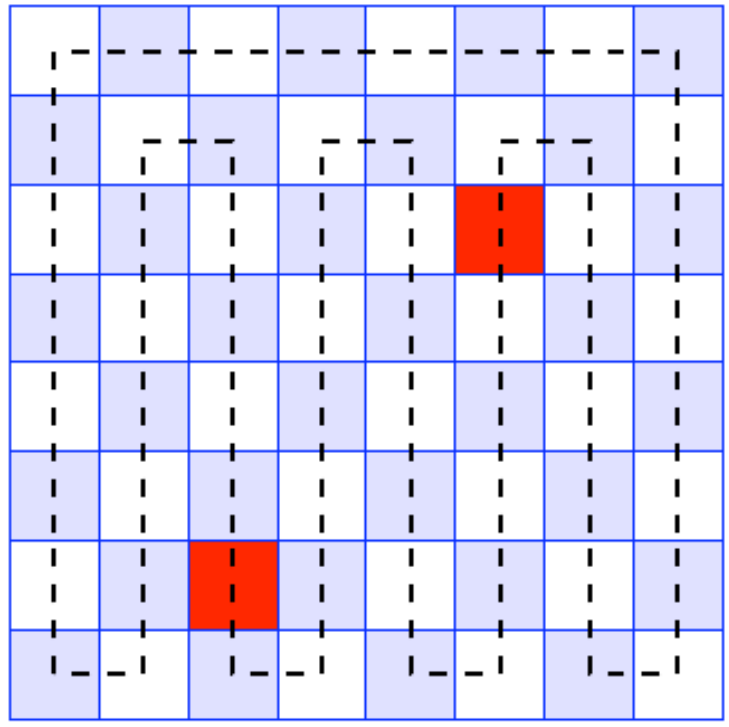

It is always possible! Draw any cycle that connects all the cells.
The length between the two red cells on this path is always even.
Tile following this path.

Tiling a fixed area

What if we remove four tiles?

Tiling a fixed area

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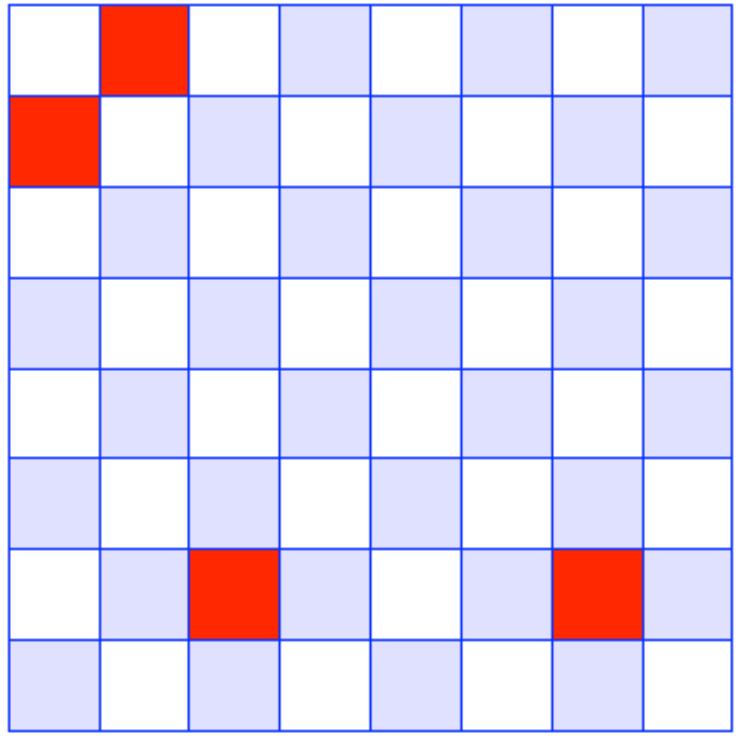
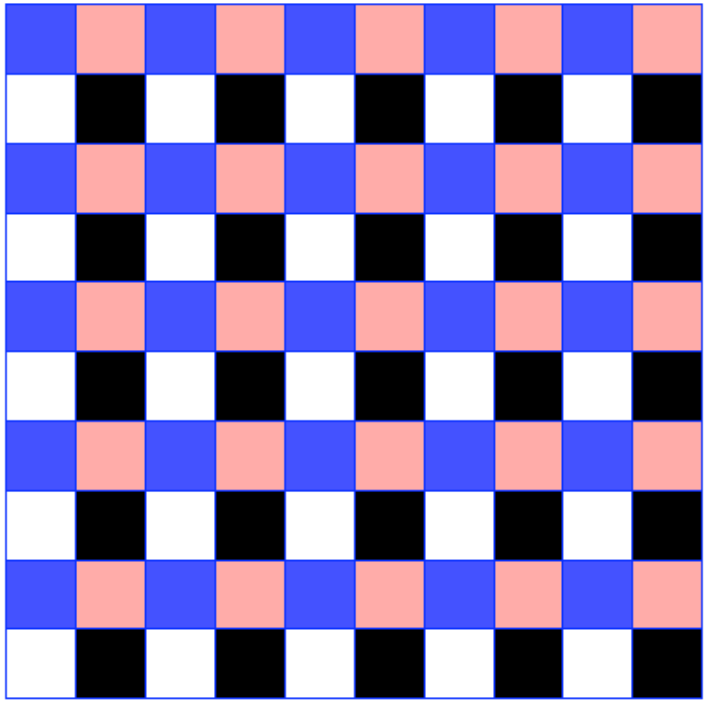


It depends... The problem has several cases, so it is not as interesting.

Tiling a fixed area

Consider a 10×10 board. Is it possible to tile it with a 1×4 rectangle?



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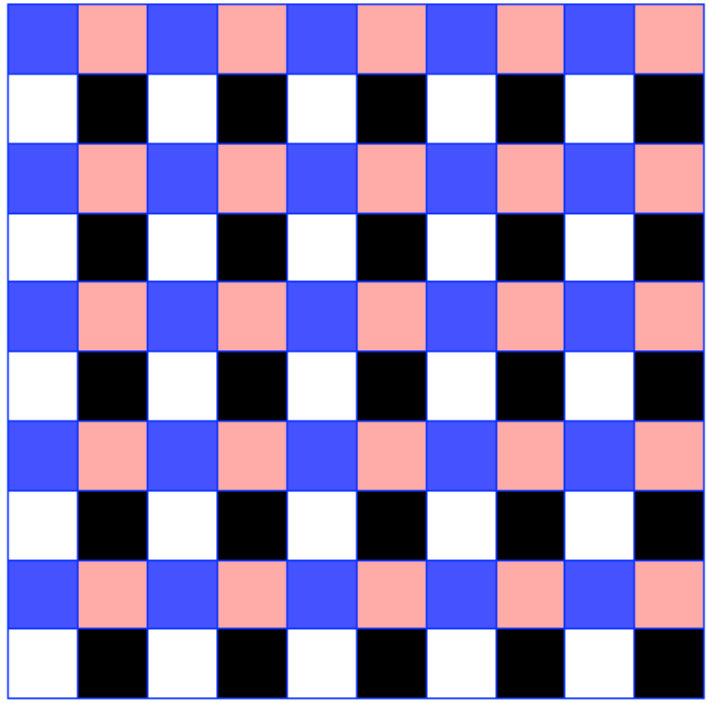
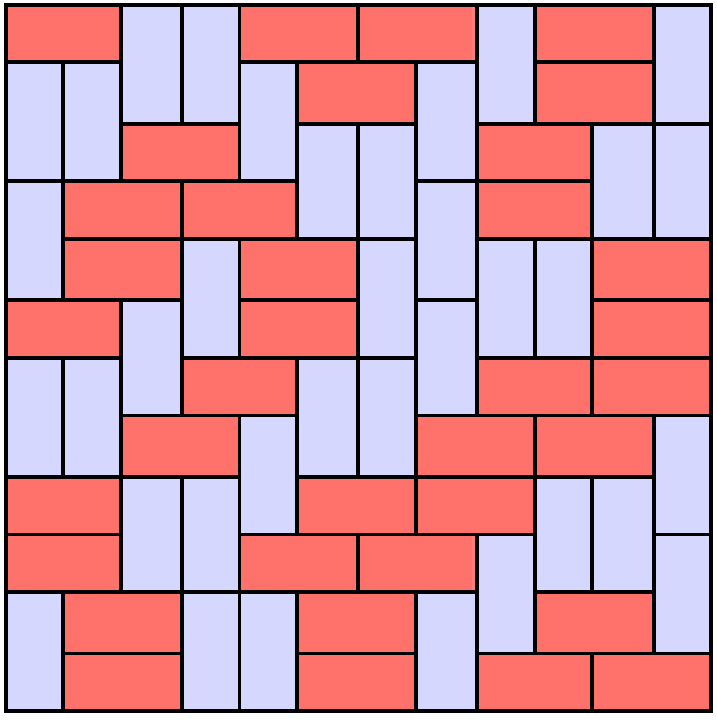
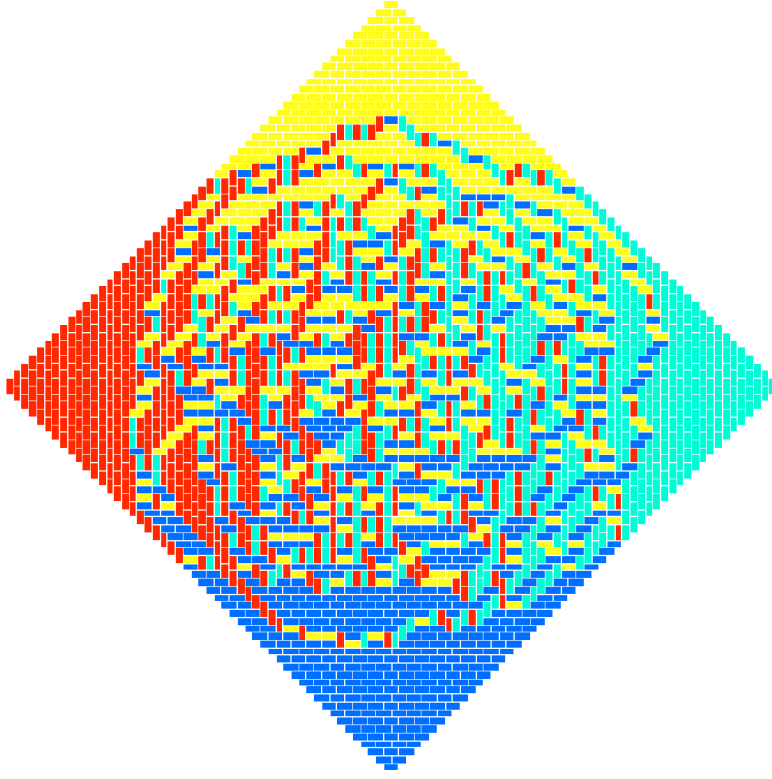


No! A tile cover two tiles of each of two colors. There are 25 tiles of each.

Aztec diamond

What does a "typical" tiling looks like?





Artistic tilings

What if we don't restrict ourselves to right angles?
Activity: In teams of 3-4, play with darts and kites, and try to build a tiling. What are your conclusions?

Artistic tilings

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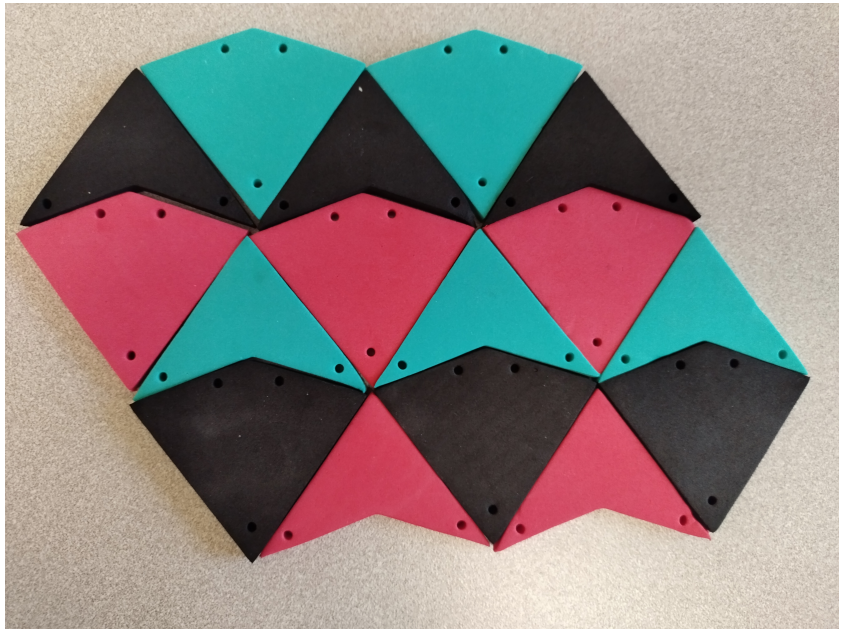
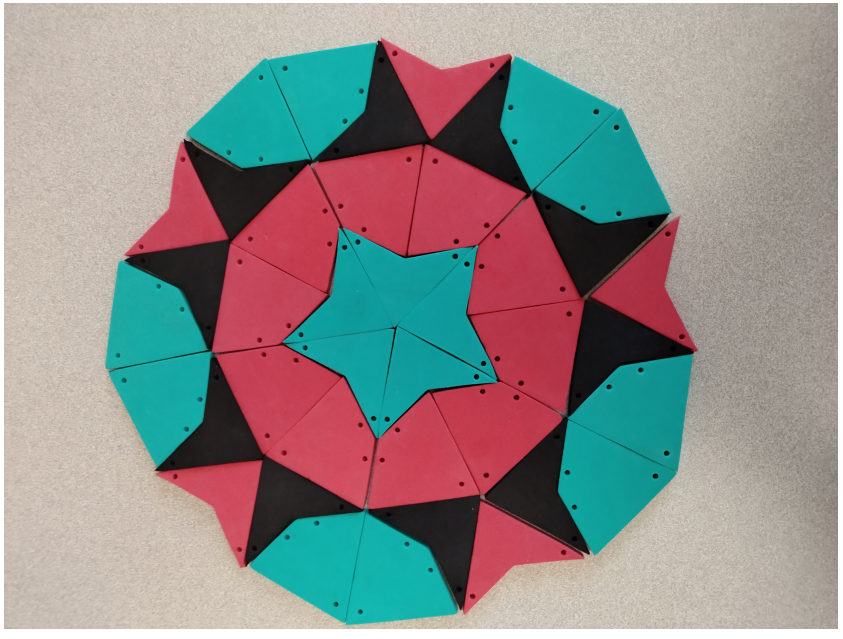
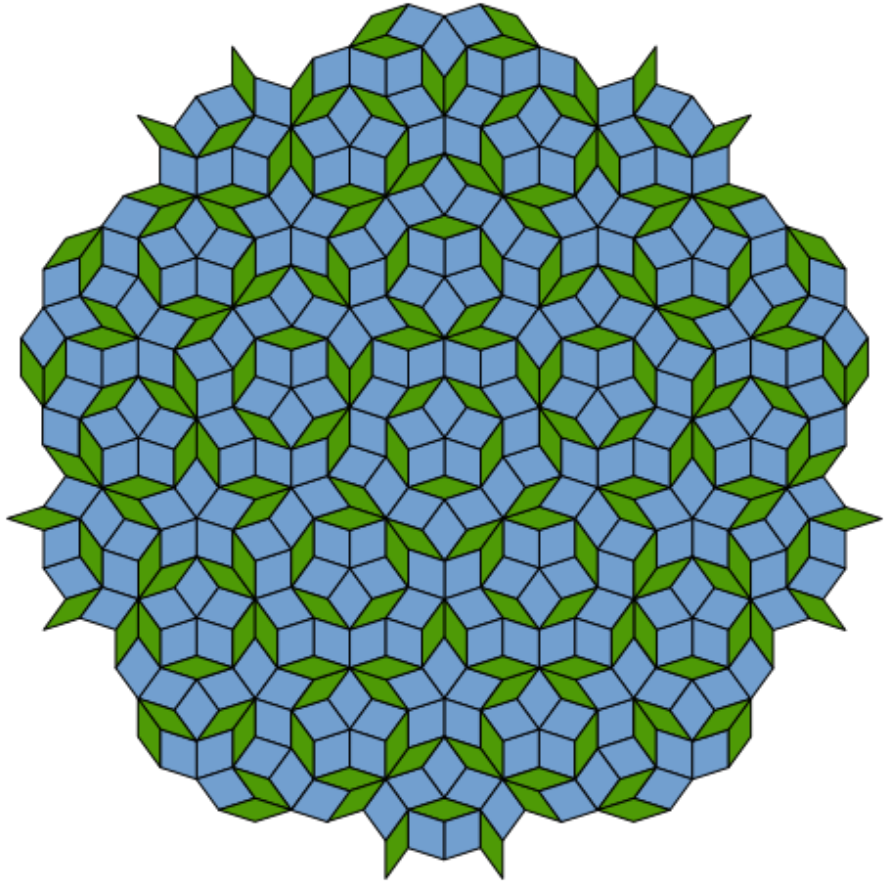
Activity: In teams of 3-4, play with darts and kites, and try to build a tiling. What are your conclusions?
Tilings can either be periodic or aperiodic:

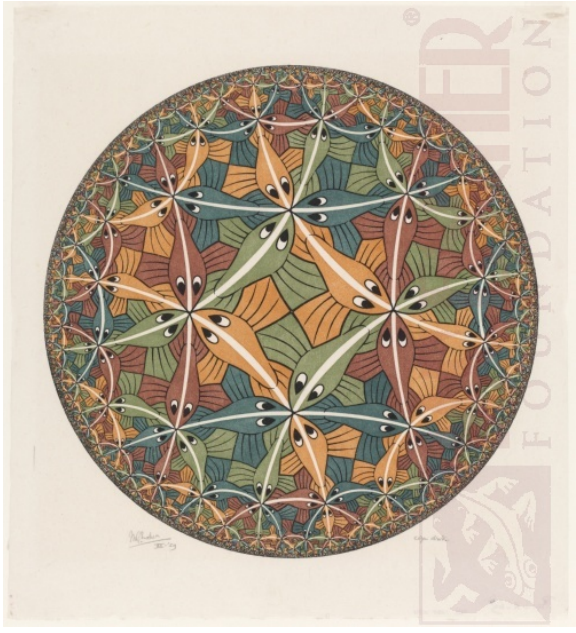




Some impressive aperiodic tilings are built using darts and kites, like the Penrose tiling:



Artistic tilings



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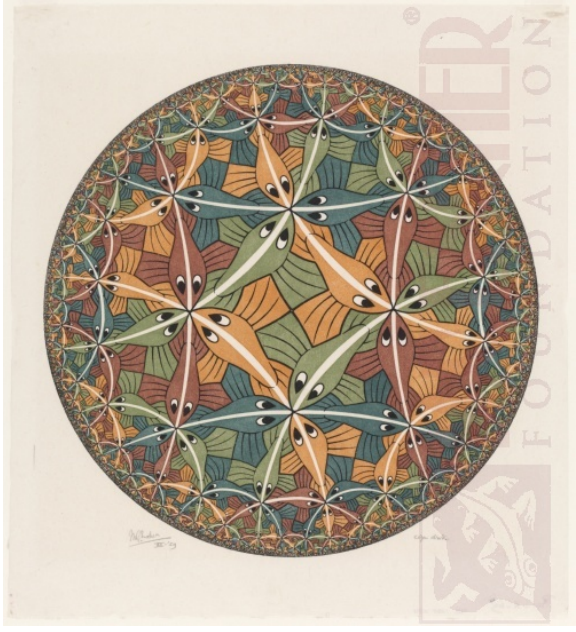
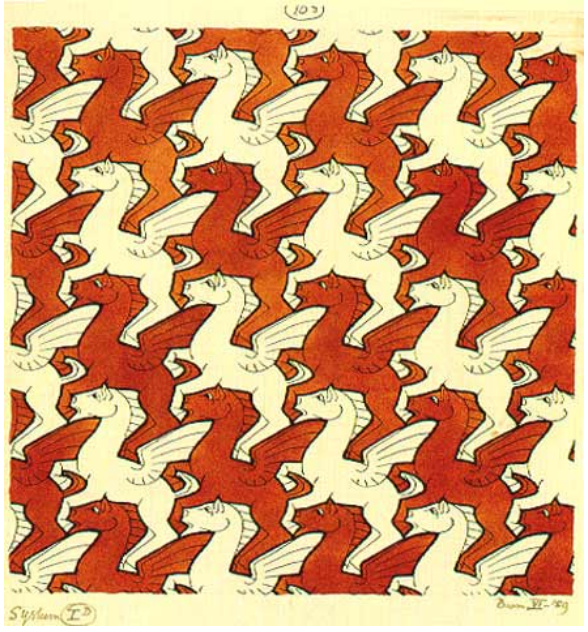
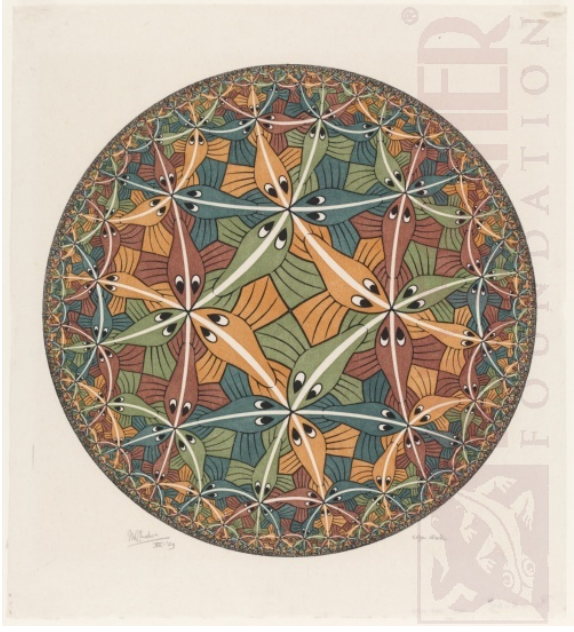




Claim: The horse tile is “a square”!

Artistic tilings



Claim: The horse tile is “a square”!

Credits

- Julia Robinson Math Festival (for the activity)
- F. Ardila, R. Stanley. *Tilings*, 2010 (for the questions on tilings chessboards and for the aztec diamond).
- Roger Penrose
- M.C. Escher