Math 25 Fall 2022

Midterm 1 – Practice

Your name: ____

INSTRUCTIONS

You may begin the exam when ready.

Write your name in the space provided above.

Use of calculators is not permitted on the exam. They are not likely to be of much help anyways.

Unless otherwise stated, you must justify your solutions to receive full credit. Work that is illegible may not be graded. Work that is scratched out will not be graded.

It is fine to leave you answer in a form such as $\ln(0.02)$ or $\sqrt{123412}$ or $(1341)^4(1231)^{-1}$. However, if an expression can be easily simplified (such as $e^{\ln(0.02)}$ or $\cos\pi$), you should simplify it.

The Honor Principle requires that you neither give nor receive any aid on this exam.

The exam has been created with the intended length of 50 minutes. It is intended to be the length of a standard midterm. This midterm is collected at the end of the X-hour.

Good luck!

Honor statement: I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

Signature: ____

Long Answer Questions

(1) (10 points) Express 1 as an integer linear combination of 101 and 90.

(2) (10 points) Determine all solutions in $\mathbb{Z}/143\mathbb{Z}$ to the equation

$$x^2 - 56 \equiv 0 \pmod{143}.$$

Note $12^2 = 144$.

(3) (10 points) Let p be a prime of the form $22k+1$ and let x be a primitive element. Prove that

$$(p+1)x^{88k} + (p+2)x^{44k} + (p-3)x^{22k} + 1 \not\equiv 0 \pmod{p}$$

(4) (10 points) Is the following statement true:

Theorem (?)**.** *Let* $n > 2$ *be an integer and let* $k := \phi(n)+1$*, where* ϕ *is the Euler totient function. Then for all* a, b *such that* $\gcd(a, n) = \gcd(b, n) = \gcd(a + b, n) = 1$*, we have* $a^k + b^k \equiv (a + b)^k \pmod{n}$*.*

If the statement is true, provide a proof. If not, provide a counter-example.

(5*) (4 points) A positive integer n is *perfect* if the sum of the positive divisors of n (including n itself) is equal to $2n$. For example, $2 \cdot 6 = 1 + 2 + 3 + 6$, so 6 is perfect.

Prove that there are no odd, squarefree, perfect numbers.

(This page is intentionally left blank in case you need extra space for any of the problems. **If you use this page for a particular problem, it is essential that you make a note on the page where the problem appears, indicating that your work is continued here.)**