NAME: ____

Math 38
Spring 2022
Midterm

Thursday, April 28

Instructions:

1. Please read all the instructions, and read and sign the statement at the bottom of this page.

2. Print your name *legibly* at the top of this page. If you are typing or sending a picture of your work, right your name at the top of the front page.

3. There are nine questions, some of which have multiple parts. Do all of them. The number of points for each question is written next to the statement. The total number of points is 100. The different parts of a problem are not necessarily worth the same number of points.

4. Except on clearly indicated problems, you must explain what you are doing, and show your work. You will be *graded on your work*, not just on your answer. Make it clear and legible, so I can follow it.

5. You have one hour to do the exam.

6. **Honor Code:** You are **not** allowed to use any material, printed or electronic. You are not allowed to discuss the problem with your peers, nor anyone other than the instructor.

In doubt about the honor code, please ask your instructor.

It is a violation of the honor code to give or receive help on this exam.

STATEMENT: I have read these instructions, and I understand how the honor code applies to this exam.

SIGNATURE: ____

HAVE YOU READ THE INSTRUCTIONS? They are on page 1.

1. (15 points)

   (a) Let T be a tree with the following vertices

   - 1 vertex of degree 2;
   - 1 vertex of degree 3;
   - 1 vertex of degree 4;
   - leaves.

   How many leaves does it have? Justify your answer.

   (b) How many labeled trees have one vertex of degree 2, one vertex of degree 3 and one vertex of degree 4, provided that all the other vertices are leaves? Justify your answer.

2. (10 points) Prove or disprove: Every Eulerian simple graph with an even number of vertices has an even number of edges.

3. (20 points) Determine the maximum number of edges in a bipartite subgraph of K_n. Prove your answer.

4. (20 points)

(a) Compute $\tau(K_{2,m})$, the number of spanning trees in $K_{2,m}$.

(b) Compute the number of isomorphism classes of spanning trees in $K_{2,m}$ (this is equivalent to counting unlabeled trees, or trees up to isomorphisms).

5. (10 points) *For this question, give a short argument to support your answer.* Let G be a graph. Then,

   (a) True or False: All spanning trees of G have the same size.

   (b) True or False: All spanning trees of G have the same diameter.

6. (5 points) *For this question, give a short argument to support your answer.* True or False: A simple graph G has a cut-edge if and only if it has no odd cycle.

7. (5 points) *For this question, give a short argument to support your answer.* True or False: If u and v are the only vertices of odd degree in a graph G, then G contains a u, v-path.

8. (10 points) Tell me at least 6 facts about the Petersen graph. You can give more than 6. (Show me your diversity of knowledge about graph theory thus far. What kinds of questions does one ask about a graph? In addition to giving properties that the Petersen graph has, it's also legitimate to list properties that the Petersen graph doesn't have, or to count things having to do with the Petersen graph. Six legitimately diverse facts, including a couple of non-trivial statements, will receive full credit; if in doubt, list more or ask.)

9. (5 points; easy credit) Tell me what you like most about graph theory so far (a favorite topic, kind of problem, way of thinking about things, etc.) or something you've learned related to graph theory (from class or not) that you enjoy a lot.

Extra room for scratch work. We will NOT look at this page, unless you write on another page “continued on page...”

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