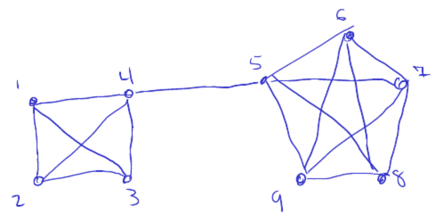
Math 38

Spring 2022

Practice Midterm Questions

1. Let $B_{n,m}$ be the graph made of the complete graphs K_n and K_m, plus one edge to join them. How many spanning trees are there in $B_{n,m}$? Your answer should be given in terms of m and n, and you should prove your solution. *Below is an example of $B_{4,5}$:*

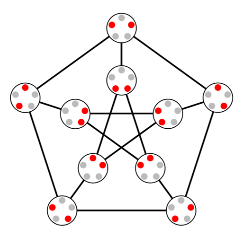


2. Prove that every simple graph with at least two vertices has two vertices of equal degree. Is the conclusion true for loopless graphs?

3. Prove or disprove: There exists a Hamiltonian graph with degree sequence $(4, 3, 2, 2, 1)$.

4. Determine the maximum number of edges in a simple graph with n vertices and an independent set of size k. Prove your answer.

5. Consider the following description of the Petersen Graph: The vertices are indexed by the 2-element subsets of $\{1, 2, 3, 4, 5\}$, and two vertices are adjacent if their intersection, as subsets, is empty. The picture below describes this definition.



   Now consider G_k with the following (similar) description: The vertices are the k-element subsets of $\{1, 2, 3, 4, 5, 6, 7\}$, and two vertices are adjacent if their intersection (as subsets) is empty. Fill out the following table. **This is a short answer problem. You don't have to justify every entry of the table.** For the column about properties, write the letter corresponding to all the properties below that apply:

   (a) Triangle-free and connected

   (b) 4-regular

(c) Complete graph

k	#vertices	#edges	$\chi(G_k)$ min. # of independent sets	Properties	Clique number largest size of clique
0					
1					
2			5		
3			3		
4					
5					
6					
7					