CS30 (Discrete Math in CS), Summer 2021

Drill 13 + 15

Topic: Probability : Basics + Conditional Probability + Bayes

Instructions

- Please submit all homework electronically in PDF, ideally typeset using LaTeX. If your handwriting is not legible, you may get 0 points.
- The drills below are supposed to be quick to do and quick to check. If a grader cannot read and understand your solution to a given drill exercise in **1 minute**, you may get a 0.
- **Collaboration Policy:** You should be able to and indeed should do the drills on your own. Collaboration is not allowed. You can ask clarification questions on Ed Discussion **privately**; the instruction team may choose to make it public. You can refer to the recommended textbook, your own course notes, posted videos, and the posted lecture notes. **Not** the web. When in doubt, consult the instructor.

Exercise 1. (**3 points**)

Let n be positive integer. Suppose you toss n different *fair* coins which fall heads with probability $1/2$ and tails with probability $1/2$. What is the probability of seeing **exactly** $\lfloor n/2 \rfloor$ heads? Your answer should be a function of n, and you should give a short reason for your answer. When n is large (say > 1000), is your answer closer to (a) $1/2$, (b) $1/\sqrt{n}$, or (c) $1/n$? Use what you learned in Drill 12.

Exercise 2. (**3 points**) We toss a fair coin independently 10 times. Let $\mathcal{E}$ be the event we see ≥ 9 heads. Let $\mathcal{F}$ be the event that we see ≥ 8 heads. What is the probability $\Pr[\mathcal{F} \mid \mathcal{E}]$? This answer should be **immediate** without any calculation. Give that reason.

What is the probability $\Pr[\mathcal{E} \mid \mathcal{F}]$? This will involve some calculation. Leave your answer as a fraction and show your work.

Exercise 3. (**4 points**)

As you all know, 0.1% of all human beings are *Wandbearers*, and they are uniformly spread out all over the world. The Ministry of Magic has trained Fluffy, the three headed dog, to sniff out Wandbearers, and it *always* detects a Wandbearer correctly. The slight nag is that Fluffy sometimes wrongly sniffs out *Muggles* (non-Wandbearers) too, but this happens only 1% of the time.

You see your friend being dragged away by the Ministry authorities after being sniffed out by Fluffy. What are the chances that she indeed is a Wandbearer?

Be precise in the events you define before you apply Bayes rule. If you just write down a fraction, you won't get any points till you define your events.