CS30 (Discrete Math in CS), Summer 2021

Drill 12

Topic: Binomial Coefficients

This drill involves coding! Yoo-hoo!?

Instructions. You have to submit this whole assignment as a **Colab Jupyter Notebook** (see here for more info); you will need a google account for this. To give us permission, you need to click "Share" (top right corner), click "Get Link", make sure you have chosen "Anyone with link can view" and **not** "Restricted". This is **important**; otherwise, we won't be able to see this, and you will not get points. To be safe, you can download your colab notebook as a .ipynb file and also submit that on Canvas.

Problem 1 (Playing with Binomial Coefficients)**.**

1. Using your bare hands figure out $\binom{10}{5}$. Show your work (how you used the formula, all the cancellations between numerator and denominator, etc). Clearly write down your answer. (**2 points**)

2. Write python code to do this: figure out $\binom{20}{10}$. I want you to write code which encodes the formula $\binom{n}{k} = \frac{n!}{k!\cdot(n-k)!}$ using the `factorial` function provided in the `math` module. So `import math`. You will need the `int`$(\cdot)$ function to convert floats into integers. Write down your answer. (**2 points**).

3. In this question I want you to figure out whether 10 divides $\binom{80}{40}$ ***without*** using a calculator/computer? Here is how you can figure this out without a computer. Note that

$$\binom{80}{40} = \frac{80!}{40!40!}$$

   How many 0s does $80!$ have in the end? Well, it is the number of factors of 5 between 1 and 80 *remembering* to count the multiples of 25 **twice**. Similarly figure out how many 0s does $40!$ have in the end. Write these answers down, and then answer whether 10 divides $\binom{80}{40}$. (**3 points**).

4. Now use the code written in part (b) and write down the last digit you obtain of $\binom{80}{40}$. Does it match with what you got in part (c)? (This may depend on the Python version you are using) (**1 point**)

5. Now write code to compute the binomial coefficients using Pascal's Theorem:

$$\text{For any positive integer } n \text{ and any } 1 \leq k \leq n, \quad \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

   Your code should have two for-loops and **NOT** be recursive. Use your code to write down the last **10** digits of $\binom{1000}{500}$. (**5 points**)

6. Using your code above, create the following plot. On the x-axis you should have the natural numbers 1 to 500. On the y-axis you should have the function

$$f(n) := \frac{\binom{n}{\lfloor n/2 \rfloor}}{(2^n/\sqrt{n})}$$

   From your plot, do you see that as n becomes larger, $f(n)$ is converging to some constant. Write down the first three digits after the decimal point of this constant. (**3 points**)

   Can you guess what this constant is? (It involves π; no worries if you can't guess it.)