CS30 (Discrete Math in CS), Summer 2021

Drill 5

Topic: Proofs

Instructions

- Please submit all homework electronically in PDF, ideally typeset using LaTeX. If your handwriting is not legible, you may get 0 points.
- The drills below are supposed to be quick to do and quick to check. If a grader cannot read and understand your solution to a given drill exercise in **1 minute**, you may get a 0.
- **Collaboration Policy:** You should be able to and indeed should do the drills on your own. Collaboration is not allowed. You can ask clarification questions on Ed Discussion **privately**; the instruction team may choose to make it public. You can refer to the recommended textbook, your own course notes, posted videos, and the posted lecture notes. **Not** the web. When in doubt, consult the instructor.

Exercise 1 (Finding Bugs!)**.**

In each of the claimed "proofs" below, mark which of the following lines have a logical error (aka bug), and write a sentence explaining why that line is wrong. (**2+2 points**)

"Claim": For any prime number q, the number $N = q! + 1$ is prime.

"Proof":

1. Let all the primes from 2 to q be called $p_1, p_2, \ldots, p_t$ where $p_1 = 2$ and $p_t = q$.
2. Observe that $N = q! + 1$ is *not divisible* by any of the primes $p_1, \ldots, p_t$. This is because each of the p_i's divide $q!$, and thus the remainder when we divide N by p_i is 1.
3. Since N is not divisible by any primes, the number N cannot be composite. Therefore, it is a prime.

"Claim": $1/8 > 1/4$.

"Proof":

1. $3 > 2$.
2. Multiplying both sides by $\log_{10}(1/2)$, we get $3\log_{10}(1/2) > 2\log_{10}(1/2)$.
3. Thus, $\log_{10}(1/2)^3 > \log_{10}(1/2)^2$.
4. That is, $\log_{10}(1/8) > \log_{10}(1/4)$.
5. This implies $1/8 > 1/4$ since $\log x > \log y$ implies $x > y$.