

# MATH 20 – HOMEWORK 1

due Wednesday, April 8th

**Instructions:** Submit HW as a pdf on Canvas. Write legibly to receive full credit. Typed solutions are also acceptable.

1. How many different ways can the letters of the word BOOKKEEPER be rearranged? (For example, the letters in the word EYE can be rearranged in three ways: EY, EYE, and YEE.)
2. Suppose that you draw a five-card hand at random from a standard deck of 52 cards. What is the probability that your hand contains two distinct pairs (for example: 3, 3, Q, Q, K or 6, 6, 9, 9, K)?
3. Suppose that  $A$  and  $B$  are events. If  $P(A \cup B) = 5/7$ ,  $P(\overline{B}) = 4/5$ , and  $P(A) = 3/5$ , then what is  $P(A \cap B)$ ?
4. (3 points) Prove that if  $A$  and  $B$  are sets, then

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

and

$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

*Hint:* One way to prove that two sets  $S$  and  $T$  are equal is to first prove that  $S \subseteq T$  and then prove that  $T \subseteq S$ . The only way both of these are true is if  $S = T$ .

5. Give a proof by contrapositive of the following statement.

*Let  $x$  and  $y$  be integers. If  $x + y$  is even then either  $x$  and  $y$  are both even or  $x$  and  $y$  are both odd.*

6. Give a proof by contradiction of the following statement.

*If  $n$  is a positive natural number and  $n^2$  is even, then  $n$  is also even.*