Math 3 Winter 2018

Midterm 2

February 15, 2018

Your name (please print): ______________________________

Circle your section:

Pauls (9L)   Ratti (10)   Malik (11)   Malik (12)

Instructions:

- This exam is closed book and closed notes. **No calculators, phones, or other devices may be used during the exam.**
- **The Honor Principle requires that you may not give or receive any help during the exam, though you may ask instructors for clarification.**
- This exam consists of **n** questions and you have **two (2)** hours to complete them. Point values are indicated for each question.
  - For multiple choice questions, no justification is required.
  - For long answer questions, you must justify all of your answers completely and show your work to receive full credit. Write clearly in complete sentences. Partial credit may be assigned to incomplete solutions.
- Please do all your work in this exam booklet. There are five (5) pages for extra work at the end of the booklet. **No work on the extra pages will be considered unless you specifically reference it on the relevant problem.**
- Good luck!

This page is for grading purposes only.

Problem	Points	Score
1-10	20	
11	10	
12	10	
13	15	
Total	**55**	

Answer Sheet for Multiple Choice Questions

Problem	Answer
1	A
2	C
3	B
4	B
5	D
6	D
7	C
8	A
9	B
10	B

Multiple Choice Questions

For problems 1-10, no justification is required and each problem is worth 2 points.

1. Let $h(x) = \sqrt{\sin(x)}$. Then, $h'(x) =$

   (a)
   $$\frac{\cos(x)}{2\sqrt{\sin(x)}}$$

   (b)
   $$\sqrt{\sin(x)}\cos(x)$$

   (c)
   $$\frac{1}{2\sqrt{\sin(x)}}$$

   (d)
   $$\sqrt{\cos(x)}$$

   (e) None of the above.

2. If $x^3y + y^2 - x^2 = 5$, what is the value of $\frac{dy}{dx}$ at the point $(2, 1)$:

   (a) $2/7$

   (b) -1

   (c) $-4/5$

   (d) $1/5$

   (e) None of the above.

3. Assume that x and y are both differentiable functions of t and satisfy $y = \sqrt{x}$. Assuming $\frac{dx}{dt} = 12$, what is $\frac{dy}{dt}$ when $x = 9$?

   (a) 1/6

   (b) 2

   (c) 3

   (d) 6

   (e) None of the above.

4. Let $g(t) = t^3 + 1$. The absolute minimum value of t on the interval $[-2, 3]$ is

   (a) 0

   (b) -7

   (c) 3

   (d) -2

   (e) None of the above.

5. Which one of these functions has no vertical asymptote?

   (a) $f(x) = \frac{x-7}{(x-7)(x-5)}$

   (b) $f(x) = \frac{x}{x^2-x-1}$

   (c) $f(x) = \frac{1}{x-2}$

   (d) $f(x) = \frac{x^2-9x+20}{(x-4)(x-5)}$

   (e) None of the above

6. Let $f(x) = \frac{1}{2}x^4 - 4x^3$. For what values of x does the graph of $f(x)$ have an inflection point?

   (a) Only at $x = 0$

   (b) Only at $x = 4$

   (c) Only at $x = 8$

   (d) At both $x = 0$ and $x = 4$

   (e) None of the above.

7. Let g be a polynomial function where $g'(x) = x^5(x+1)(x-1)$. At how many points does the graph of g have a relative minimum?

   (a) None

   (b) One

   (c) Two

   (d) Three

   (e) None of the above.

8. Let

$$f(x) = \frac{\sin(x^2)}{3x}.$$

   What is $f'(x)$?

   (a)

$$\frac{6x^2\cos(x^2) - 3\sin(x^2)}{9x^2}$$

   (b)

$$\frac{2x\cos(x^2)}{3}$$

   (c)

$$\frac{2\cos(x^2)}{3}$$

   (d)

$$\frac{2x^2\sin(x^2) - \cos(x^2)}{3x^2}$$

   (e) None of the above.

9. What is a correct statement of the Extreme Value Theorem?

   (a) If $f(x)$ is differentiable on the closed interval $[a, b]$ then there are points c, d in the interval $[a, b]$ where $f(x)$ attains is absolute maximum and absolute minimum.

   (b) If $f(x)$ is continuous on the closed interval $[a, b]$ then there are points c, d in the interval $[a, b]$ where $f(x)$ attains is absolute maximum and absolute minimum.

   (c) If $f(x)$ is continuous on the closed interval $[a, b]$ then there are points c, d in the interval $[a, b]$ where $f(x)$ attains is relative maximum and relative minimum.

   (d) If $f(x)$ is differentiable on the closed interval $[a, b]$ then there are points c, d in the interval $[a, b]$ where $f(x)$ attains is relative maximum and relative minimum.

   (e) None of the above.

10. At what value of x does $x \ln(x)$ reach its absolute minimum?

   (a) 0

   (b) $1/e$

   (c) e

   (d) 1

   (e) None of the above.

Long Answer Questions

For each long answer problem, you must give a complete explanation and provide full justification of your answer to receive full credit. Write clearly in complete sentences.

11. (10 points total) A triangle has a height that is increasing at a rate of 2 cm/sec and its area is increasing at a rate of 4 cm^2/sec. Find the rate at which the base of the triangle is changing when the height of the triangle is 4 cm and the area is 20 cm^2.

 Solution: The area of a triangle relates its base b and height h through the formula $A = \frac{1}{2}bh$. The given information tells us that $\frac{dh}{dt} = 2$ and $\frac{dA}{dt} = 4$. We wish to solve for $\frac{db}{dt}$. Differentiating both sides of the equation using the product and chain rules yields:

$$\frac{dA}{dt} = \frac{1}{2}\left(\frac{db}{dt}h + b\frac{dh}{dt}\right).$$

 Solving for $\frac{db}{dt}$ gives:

$$\frac{db}{dt} = \frac{2\frac{dA}{dt} - b\frac{dh}{dt}}{h}.$$

 When the height is 4 and the area is 20, we have that $20 = 2b$, or $b = 10$. Putting all this together, we have

$$\frac{db}{dt} = \frac{2 \cdot 4 - 10 \cdot 2}{4} = \frac{8 - 20}{4} = \frac{-12}{4} = -3\ cm/sec.$$

12. (10 points) If $\cos^2(x) + \cos^2(y) = \cos(2x + 2y)$, find $\frac{dy}{dx}$ assuming y is a function of x.

Solution: We have

$$\cos^2(x) + \cos^2(y) = \cos(2x + 2y).$$

On differentiating both sides of the equation, we get

$$D(\cos^2(x) + \cos^2(y)) = D(\cos(2x + 2y)),$$

$$D(\cos^2(x)) + D(\cos^2(y)) = D(\cos(2x + 2y)).$$

Using power rule and chain rule, we get

$$2\cos(x)D(\cos(x)) + 2\cos(y)D(\cos(y)) = -\sin(2x + 2y)D(2x + 2y)$$

Again, using chain rule, we have

$$2\cos(x)(-\sin x) + 2\cos(y)(-\sin(y)y') = -\sin(2x + 2y)(2 + 2y').$$

Now solve for y',

$$-2\cos(x)\sin(x) - 2\cos(y)\sin(y)y' = -2\sin(2x + 2y) - 2y'\sin(2x + 2y).$$

Factor out y',

$$y'(-2\cos(y)\sin(y) + 2\sin(2x + 2y)) = 2\cos(x)\sin(x) - 2\sin(2x + 2y),$$

$$y' = \frac{2\cos(x)\sin(x) - 2\sin(2x + 2y)}{-2\cos(y)\sin(y) + 2\sin(2x + 2y)}.$$

13. (15 points)

Consider the function $h(x) = \dfrac{\sqrt{2-3x^2}}{x}$. Our goal in this problem is to sketch its graph.

(a) What is the domain of $h(x)$?

Solution: $h(x)$ is undefined at $x = 0$, as this is a zero of the denominator, and on $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$, as the term under the square root is less than zero. Thus, the domain of $h(x)$ is $[-\sqrt{2/3}, 0) \cup (0, \sqrt{2/3}]$.

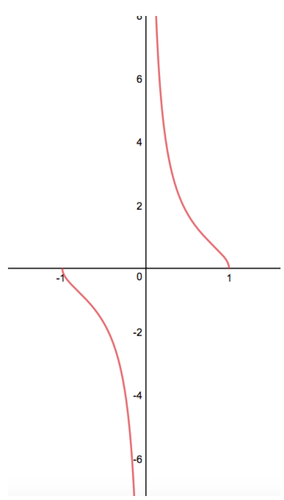
(b) Describe any vertical and horizontal asymptotes of $h(x)$. Remember to justify your answers!

Solution: As $h(x)$ is undefined on $(-\infty, -\sqrt{2/3}) \cup (\sqrt{2/3}, \infty)$, it doesn't have any horizontal asymptotes. We claim $\lim_{x\to 0^-} h(x) = -\infty$. Since $\sqrt{2-3x^2}$ is a composition of a polynomial and the square root function, both of which are continuous on their domains, it tends to 1 as x tends to zero by direct substitution. Since the denominator tends to zero through negative numbers, the ratio tends to $-\infty$. Similarly,$\lim_{x\to 0^+} h(x) = \infty$. Hence, $h(x)$ has a vertical asymptote at $x = 0$.

(c) What are the intervals on which h is increasing/decreasing?

Solution: For $h(x) = \dfrac{\sqrt{2-3x^2}}{x}$, $h'(x) = \dfrac{-2}{x^2\sqrt{2-3x^2}}$ using the quotient rule, chain rule, and power rules. As the domain of $h'(x)$ is $(-\sqrt{2/3}, 0) \cup (0, \sqrt{2/3})$, Further, $h'(x)$ is never zero and is undefined at $x = 0, \pm\sqrt{2/3}$. Checking points in the intervals defined, we have (for example) $h'(1/2) < 0$ and $h'(-1/2) < 0$. Thus, $h(x)$ is decreasing on its entire domain.

(d) Using the information in the previous parts of the problem, sketch the graph as best you can. Solution:



(e) What features of your sketch are not completely justified by the work in parts (a)-(c)? What further computations would you perform to justify or correct these?

Solution: There are a number of ways to answer this correctly based on how you approached the problem. The main features of the graph that we see but haven't justified are concavity, possible inflection points, and intercept positions. We can nail these down using the second derivative to find concavity information and calculating the x-intercepts.

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